Rayleigh scattering in high-\(Q\) microspheres

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Received July 1, 1999; revised manuscript received January 19, 2000

The Rayleigh scattering has to be largely suppressed in high-\(Q\) whispering-gallery modes in microspheres because of restrictions imposed on scattering angles by cavity confinement. Earlier estimates of the fundamental limit for the quality factor in fused-silica microspheres are revisited, and \(Q \sim 10^{12}\) is predicted in few-millimeter-size fused-silica spheres, if the surface hydration problem is overcome. Particular effects of surface scattering losses are analyzed, and the manifestation of scattering in the form of intermode coupling is calculated. The predominant effect of counterpropagating mode coupling (intracavity backscattering) is analyzed in the presence of a mode-matched traveling-wave coupler. As much as 100% resonance reflection regime is shown to be feasible. © 2000 Optical Society of America [S0740-3224(00)00406-9]

OCIS codes: 270.0270, 290.0290.

1. INTRODUCTION

Optical microsphere resonators\(^1\) with whispering-gallery (WG) modes, with their unique combination of a very high quality factor (as great as \(10^{10}\)) and submillimeter size, are attractive new building blocks for applications in optoelectronics and measurement science. Currently, microspheres are prepared by the fusion of silica or other glass preforms in the flame of a microburner or in the \(\text{CO}_2\) laser beam. Microspheres can be, for example, used as interferometers and filters with record finesse,\(^2,3\) as ultrasmall volume photon traps in cavity QED experiments,\(^1,4,5\) and as a compact and efficient tool for laser frequency locking\(^6\) (the last application allows the creation of ultracompact inexpensive tunable lasers with a sub-kHz linewidth).

Basic electrodynamics theory of WG modes is given in classical monographs (see, for example, Ref. 7). The high-\(Q\) resonances in spherical dielectric particles have been extensively investigated in the past two decades in connection with scattering in aerosols.\(^8\) Solid microspheres as functional microcavities attracted an increasing interest in recent years and prompted a number of theoretical and experimental studies focused on the properties of highly confined WG modes, methods of effective evanescent wave coupling, and different applications.

For most applications, the crucial motivation factor is a very high \(Q\) factor of WG modes, ultimately limited by optical attenuation in the cavity material. Mechanisms that limit the \(Q\) of microspheres, such as bending losses, scattering on residual surface inhomogeneities, and coupling losses, have been outlined in several papers. Recent experiments have shown that in the absence of ambient absorbers (including even a single monolayer of water chemosorbed on the surface), quality factors as high as \(Q = 8 \times 10^9\)—very close to the fundamental limit defined by material attenuation—can be obtained in the visible and the near-IR bands.\(^9,3\) Experiments are currently underway to investigate the possibility of achieving ever higher \(Q\) closer to the minimum of attenuation of fused silica, 1.55 \(\mu\)m.

In all previous estimates of the quality-factor limitation, it was implied that the material-related losses in microspheres can adequately be modeled by the same plane-wave attenuation coefficient that is known for plane waves in the bulk medium and has been measured by fiber-optic methods. This is not, however, a straightforward assumption, because one of the two basic components of loss—Rayleigh scattering—can be modified inside the three-dimensional cavity. The scattering effects in microspheres were formerly analyzed incompletely. Scattering leads not only to the limitation of the \(Q\) factor but also to the intermode coupling. Most significant, it couples initially degenerate counterpropagating modes in the spheres and creates the intracavity feedback mechanism instrumental for the laser frequency locking application. In the frequency domain, intracavity backscattering is observed as the splitting of initially degenerate WG mode resonances and the occurrence of characteristic mode doublets. This effect was first observed and reported briefly in Ref. 10 and later addressed in more detail\(^11\) that included, along with experimental data, initial semiqualitative estimates of the splitting. More recently, the doublet structure of WG modes that are due to backscattering was identified in Ref. 6, where this effect was used as a source of resonance optical feedback for narrowing the linewidth of a semiconductor laser. Except for these basic estimates, no explicit analysis was done for backscattering in microspheres proper and for the parameters of resonance feedback in evanescent couplers. The least elaborated estimate remained analysis of surface scattering effects. It was best illustrated by discrepancies in both numerical and analytic estimates of...
scattering losses as functions of resonator size and wavelength of probing light.\textsuperscript{1,3,9,12}

In this paper we present a complete analysis of scattering effects in high-\(Q\) microsphere cavities. The two goals of the calculations are (1) to analyze modification of the Rayleigh scattering losses in microspheres (both by volumetric and surface inhomogeneities) and reassess the fundamental limit for the quality factor and (2) to analyze formation of the intracavity backscattered wave (in the presence of an evanescent coupler) and calculate explicit parameters of the backscattering doublet.

2. SCATTERING BY INTERNAL THERMODYNAMICAL INHOMOGENEITIES AND THE QUALITY FACTOR OF MICROSPHERES

Intrinsic scattering and absorption losses in microresonators were previously estimated from the bulk losses as

\[ Q = \frac{2\pi n}{(\alpha \lambda)}, \]

where \(n\) is the index of refraction, \(\alpha\) is the intensity attenuation coefficient, and \(\lambda\) is the wavelength. It implies that linear attenuation for whispering-gallery modes, considered as closed resonant waves circulating in the sphere, is identical to that of plane waves in the bulk medium. However, this approach is not quite accurate for scattering. Let us review here the method of derivation of scattering coefficient \(\alpha\) (see, for example, Ref. 13) to see what modifications should be made in Eq. (1) to take into account specific features of the microspheres.

Let us divide the whole volume of the medium into small volumes \(dv\), each having, owing to fluctuations, dielectric constant \(\epsilon(r) = \delta\epsilon(r) + \epsilon^0\). Internal small inhomogeneities in the field of the mode behave as dipoles that re-emit light in all directions according to the Rayleigh formula,

\[ I_s \sim \frac{\pi^2 \sin^2 \theta}{\lambda^4 r^2} \left( \int \int \delta\epsilon(r_1) \delta\epsilon(r_2) dv_1 dv_2 \right), \]

where \(\theta\) is the angle between the dipole axis (coinciding with polarization of field) and direction of scattering and \(r\) is the distance from the scatterer.

The next step is to integrate Eq. (2) over all angles on a large sphere \((r \rightarrow \infty)\) to obtain total power of scattering:

\[ P_s = \int \int \frac{8\pi^2}{3\lambda^4} \delta\epsilon(r_1) \delta\epsilon(r_2) dv_1 dv_2. \]

However, for the microsphere this approach is not correct. We should take into account the total internal reflection.

Within the quasi-geometrical approach, we have to assume that the beams falling on the surface under the angle larger than critical one, \(\gamma_0 > \arcsin(1/n)\), will (1) be fed back into the mode (if this angle lies inside the mode's caustic), (2) be suppressed in destructive interference during several reflections, or (3) be fed into the caustic of another mode (overlapping with the original one in frequency domain)—thus leading to intermode coupling. Because of the rareness of mode spectrum and a very high quality factor, the third effect can be neglected (the specific case of coupling between oppositely circulating de-

generate modes is separately analyzed below). Therefore only beams that fall under angles less than critical one should be added to scattering losses. We should also disregard here partially reflected beams under \(\gamma_0 > \arcsin(1/n)\) because they leave the resonator after a few reflections. Conditions for the cutoff angles for TE and TM modes are the following,

\[ \sin^2 \gamma_{TE} = \left( \frac{a - d}{a} \right)^2 \left( 1 - \sin^2 \theta \cos^2 \varphi \right) < \frac{1}{n^2}, \]

\[ \sin^2 \gamma_{TM} = \left( \frac{a - d}{a} \right)^2 \sin^2 \theta < \frac{1}{n^2}. \]

where \(d\) is the distance of the dipole from the surface and \(a\) is the radius of the microsphere. If \(d < a\) (which is always correct for high-\(Q\) whispering-gallery modes, the first terms in Eqs. (4) may be omitted, and hence the result will not depend on the size of the resonator. We skip here further derivation of \(\alpha\) by calculating thermodynamical calculations, since the conditions of angular cutoff do not interfere with it.\textsuperscript{13} (Rigorously speaking, in very small spheres even thermodynamic calculations have to be modified owing to the discreteness of the phonon spectrum. In our case, however, with a diameter of a few hundred micrometers, bulk thermodynamics stands.) The plane-wave scattering in the bulk is\textsuperscript{13}

\[ \alpha_{is} = \frac{8\pi^3}{3\lambda^4} n^2 p^2 \kappa T \beta_T, \]

where “is” denotes internal scattering, \(\kappa\) is the Boltzmann constant, \(T\) is the effective temperature of glassification (~1500 K for fused silica), \(\beta_T\) is the isothermal compressibility, and \(p\) is the Pockels coefficient of optoelasticity at this temperature.

Angular cutoff conditions may be taken into account by the introduction of suppression coefficients:

\[ Q_{is} = K_{TE,TM} \frac{2\pi n}{\alpha_{is} \lambda}. \]

This coefficient \(K_{TE,TM}\) is equal to the ratio of complete scattered power over the power scattered into the angles satisfying conditions (4).

Numerical calculations of the integrals with account of angular cutoff for fused-silica microspheres \((n = 1.45)\) yield

\[ K_{TE} = 2.8, \quad K_{TM} = 9.6. \]

This result indicates that up to nearly one-order-of-magnitude suppression of bulk scattering losses can be expected in microspheres, compared with plane-wave propagation in continuous silica. Maximum suppression is predicted for TM modes. As follows from further calculations, however, TM modes have a larger field on the interface and therefore have larger losses that are due to scattering by surface inhomogeneities and absorption by surface contaminants.
3. SCATTERING BY SURFACE INHOMOGENEITIES

To analyze the surface scattering, by analogy with Section 2, we shall calculate the modified value of linear attenuation \( \alpha_{ss} \), that describes losses of traveling wave per unit length. We can start with the same expression in integral form (2) but now shall take into account only surface inhomogeneities. As before, we should integrate this expression over angles with an account of the total internal reflection, but for surface dipoles the part scattered to outside the sphere is all loss. From the symmetry of the integrals, the suppression coefficient for scattering loss due to angular cutoff can be expressed through the above-calculated bulk scattering suppression coefficients as \( 2K_{TE, TM}/(K_{TE, TM} + 1) \). As before, let us start with the calculation of linear attenuation that is due to surface scattering.

Let the wave with intensity distribution \( I(y, z) \) travel along a guiding surface parallel to the local \( x \) axis; the \( y \) axis is in the plane of the surface, and the \( z \) axis is orthogonal to the surface. Small surface roughness can be modeled by the inhomogeneity of the dielectric constant,

\[
\delta\varepsilon(x, y, z) = (\varepsilon_0 - 1)f(x, y)\delta(z),
\]

where \( \delta(z) \) is a delta function. If surface inhomogeneities are weakly correlated and their correlation function quickly tends to zero at distances much smaller than the wavelength of light, roughness may be described by only two parameters: variance \( \sigma = [(f(x, y))^2]^{1/2} \) and correlation length \( B \). In this case

\[
P_s = \int I(y, 0) \frac{16\pi^2}{3\lambda^4} (n^2 - 1)\pi B^2\sigma^2 dy = P\sigma dx.
\]

Thus considering that the power of the wave is equal to \( P = \int I(y, z) dy dz \) and considering that the wave travels close to the surface, we obtain

\[
\alpha_{ss} = \frac{I(y, 0) 16(n^2 - 1)\pi B^2 \sigma^2}{16\pi^2/3\lambda^4} = \frac{I(y, 0)}{3\lambda^4},
\]

Finally, we obtain expression for the quality-factor limitation by surface scattering losses:

\[
Q_{ss} = \frac{K_{TE} 3\lambda^3 \alpha}{1 + \frac{K_{TE} 3\lambda^3 \alpha}{8n\pi B^2 \sigma^2}}.
\]

This expression is different from that given in Ref. 3, the reason being the underestimated volumetric ratio (11) in Ref. 3: \( \alpha \) being proportional to \( \sqrt{\alpha/\lambda} \) and not \( \alpha \). The same paper presents the results of an atomic force microscopy study of a fused-silica microsphere surface, with estimated values \( B = 5 \text{ nm} \) and \( \sigma = 1.7 \text{ nm} \). In Fig. 1 we present the modified prediction for feasible \( Q \) values of fused-silica microspheres, with account of the modified scattering losses. The \( Q \)-versus-wavelength plot incorporates the literature data\(^{13,15} \) on UV and IR absorption components of losses in fused silica. According to these literature data, bulk losses in continuous fused silica can be approximated as follows:

\[
\alpha = [0.7 \mu m^4/\lambda^4 + 1.1 \times 10^{-3} \exp(4.6 \mu m/\lambda)] dB/km.
\]

For the estimates of surface scattering, the sphere radius of \( a = 1 \text{ mm} \) was taken (Fig. 1), and the surface-scattering-limited \( Q \) factor scales linearly with the radius. Therefore, in the visible range, the predicted suppression of bulk losses is compensated for submillimeter spheres by surface scattering, and values higher than \( Q = 8 \times 10^9 \) demonstrated in an experiment with 600–800 \( \mu \)m spheres may not be expected. At longer wavelengths, however, much larger \( Q \)’s can be expected than was earlier predicted. For very large spheres (several millimeters in diameter), quality factors sufficiently higher than \( 10^{11} \) can be obtained, and \( Q = 10^{12} \) may be expected as Rayleigh scattering can reportedly be further lowered by heat treatment, at least by 25%\(^{16} \). [An earlier prediction for the fundamental maximum of \( Q \) in microspheres was \( Q = 1.5 \times 10^{11} \) (Ref. 1).]

Experimental demonstration of the predicted ultrahigh quality factors in microspheres requires that extrinsic loss mechanisms be avoided. One of those is optical absorption by chemosorbed layers of \( \text{OH}^- \) ions and water. According to Refs. 3 and 9, in the visible range at 633 nm in normal laboratory conditions, deposition of an ~1.5 \( \mu \)m water monolayer during the first ~100 s from sphere fabrication results in a drop of quality factor from \( Q = 8 \times 10^{11} \) to ~\( 8 \times 10^{10} \).
The dielectric constant may be written in the form

$$\frac{\kappa T}{\bar{\sigma}(L - 1)(L + 2)},$$

(15)

where $\bar{\sigma}$ is the surface-tension coefficient $\sim 200$ dyn/cm for fused silica at temperature $T = 1500$ K. As estimates show, the size of these fluctuations will be several times less than was measured in Ref. 3. However, it is worth mentioning that the correlation function, calculated for such inhomogeneities, has a logarithmic shape and therefore may not be characterized by correlation length, and another approach has to be developed to calculate surface losses with better precision.

4. MODE COUPLING BY INHOMOGENEITIES: A GENERAL APPROACH

Coupling of modes in microspheres because of scattering on surface and internal inhomogeneities may be described with the variational approach. Random deviations of the dielectric constant may be written in the form

$$\delta \varepsilon = f(\theta, \phi) F(r),$$

(16)

where $F(r)$ is the random radial function and $f(\theta, \phi)$ is the random angular function. In the particular case of small surface roughness, random fluctuations of the surface of the sphere may be described as

$$r(\theta, \phi) = a + f(\theta, \phi),$$

(17)

and expression (16) may be written as

$$\delta \varepsilon = (n^2 - 1)f(\theta, \phi) \delta (r - a).$$

(18)

From the Maxwell equation, the wave equation for the fields inside the microsphere with inhomogeneities can be obtained, as follows:

$$\Delta E - \frac{\varepsilon_0'(r)}{c^2} \frac{\delta \varepsilon(r)}{c^2} \frac{\partial^2 E}{\partial t^2} = 0.$$  (19)

The solutions of an unperturbed equation without inhomogeneities (if $\delta \varepsilon = 0$) have the form

$$E_j = \exp(-i\omega t) e_j(r, \theta, \phi),$$

(20)

where $e_j(r, \theta, \phi)$ is the vector harmonic that satisfies the Helmholtz equation,

$$\Delta e_j + \varepsilon_0 k_j^2 e_j = 0,$$

(21)

index $j$ corresponds to all possible types of oscillations, and $j = 0$ corresponds to the initially excited mode. Using the method of slowly varying amplitudes, we find the solution as

$$E = \exp(-i\omega t) \sum A_j(t) e_j.$$  (22)

After substituting this sum into Eq. (19) and omitting small terms, we obtain

$$2i \omega_0 \varepsilon_0 \sum \frac{dA_j(t)}{dt} e_j + \omega_0^2 \delta \varepsilon \sum A_j(t) e_j$$

$$+ \varepsilon_0 \sum_j (\omega_j^2 - \omega_0^2) A_j(t) e_j = 0.$$  (23)

After multiplication of this equation on $e_j^*$ and integration over the whole volume, with account of the modes’ orthogonality, we obtain the usual equations for coupled modes:

$$\frac{dA_k}{dt} + i \Delta \omega_k A_k = i \sum_j A_j \beta_{jk},$$

(24)

where $\Delta \omega_k = \omega_k - \omega_0$.

$$\beta_{jk} = \frac{\omega_0}{2n^2} \int |e_j|^2 |e_k|^2 dv.$$  (25)

In this expression it is the random function $\delta \varepsilon$ that leads to the coupling between $e_j$ and $e_k$. We are interested only in the modulus of the coefficient $\beta_{jk}$, which determines the rate of energy transfer between the modes. If the value of inhomogeneities and their correlation length are very small compared with the wavelength, we can average $\beta_{jk}^2$ to find that

$$\beta_{jk}^2 = \frac{\omega_0}{4n^2} \left( \frac{\int |e_j(r)|^2 |e_k(0)|^2 dv}{V_{jk}} \right),$$

(26)

where $V_{jk}$ is an overlap volume of modes:

$$V_{jk} = \frac{\int |e_j|^2 dv \int |e_k|^2 dv}{\int |e_j|^2 |e_k|^2 dv}.$$  (27)

In the most interesting case of coupling between the two modes $A_+(t)$ and $A_-(t)$, traveling inside the microsphere in opposite directions, the field distributions for these two modes differ only by phase factor $\exp(\pm im\phi)$. In this case $e_j = e_j^*$ and $V_{jk}$ transform into an effective volume of field localization:
5. MODE COUPLING BY INHOMOGENEITIES: BACKSCATTERING AND THE RESONANCE FEEDBACK IN AN EVANESCENT COUPLER

To analyze the consequences of internal coupling between counterrotating modes on the output characteristics of a resonator, we can use the same quasi-geometrical approach that we used in Ref. 14 to analyze coupler devices for the microspheres. For simplicity, we analyze here only the case of an ideally mode-matched (or monomode) coupler device. The set of equations for the internal and external amplitudes looks as follows:

\[
\frac{dA_+}{dt} + (\delta_0 + \delta_c + i\Delta\omega)A_+ = iA_-\beta + \frac{T}{\tau_0}B_{in},
\]

\[
\frac{dA_-}{dt} + (\delta_0 + \delta_c + i\Delta\omega)A_- = iA_+\beta,
\]

\[
B_t = \sqrt{1 - T^2}B_{in} + iTA_+, \quad B_r = iTA_-,
\]

where \(A_+(t)\) and \(A_-(t)\) are, as before, the amplitudes of oppositely circulating modes of the total internal reflection in the resonator (Fig. 2) to model the whispering-gallery modes. \(B_{in}\) is the amplitude of pump, and \(B_t\) and \(B_r\) are output amplitudes transmitted and reflected in the coupler. \(T\) is the amplitude transmittance coefficient, describing the coupler; \(\delta_0 = 2\pi\eta/\alpha\lambda\) is the decrement of internal losses; \(\delta_c = T^2/2\tau_0\) is the decrement of the coupler device; \(\tau_0\) is the circulation time \(\tau_0 = 2\pi\alpha l/c\); and \(\Delta\omega\) is frequency detuning from unperturbed resonance frequency \(\omega_0\) (for details, see Ref. 9). The stationary solution of Eqs. (29) is the following:

\[
A_+ = i\frac{2\delta_c\beta}{T(\delta_0 + \delta_c)^2 + \beta^2 - \Delta\omega^2 + i2\Delta\omega(\delta_0 + \delta_c)}B_{in},
\]

\[
A_- = -\frac{1}{T(\delta_0 + \delta_c)^2 + \beta^2 - \Delta\omega^2 + i2\Delta\omega(\delta_0 + \delta_c)}B_{in},
\]

This means that ringdown time is equal to the time needed to repump the circulating mode in an oppositely circulating one.

The case of \(\beta \gg \delta_0 + \delta_c\) is much more interesting and even leads to somewhat unexpected results. In this case there are two resonances at frequencies \(\Delta\omega = \pm[\beta^2 - (\delta_0 + \delta_c)^2]^{1/2}\), i.e., internal coupling lifts the degeneracy between sine and cosine standing modes in the microsphere, and there is enough time to form them. All intensities in resonances in this case do not depend on \(\beta\):

\[
|A_+|^2 = |A_-|^2 = \frac{1}{T^2}(\delta_c^2 B_{in}^2),
\]

\[
|B_t|^2 = \frac{\delta_0^2}{(\delta_0 + \delta_c)^2}B_{in},
\]

\[
|B_r|^2 = \frac{\delta_c^2}{(\delta_0 + \delta_c)^2}B_{in}.
\]
What is interesting and not evident intuitively is that if \( \delta_0 < \delta_c \) (overcoupling) but still \( \delta_0 + \delta_c < \beta \), the main part of input power is backscattered, whereas transmitted power tends to zero. All these regimes are clearly seen in Fig. 3. This property may be valuable for future applications of microspheres in laser stabilization. To verify this result, additional experiments are required. In our preliminary experiments, we observed repeatedly that backscattering was absent when there was no splitting and also that the backscattered power weakly depended on loading, whereas doublets were clearly seen. Similar observations was reported in Ref. 11.

6. MODE COUPLING BY INHOMOGENEITIES: PARAMETERS OF THE NORMAL MODE SPLITTING

Mode coupling leads to resolved splitting of initially degenerate modes: If \( \beta \) constant is much larger than the mode decrement of internal and coupling attenuation, \( \delta_0 + \delta_c \), then

\[
\frac{\Delta \omega}{\omega} = \frac{2\beta}{\omega_0}. \tag{33}
\]

If thermodynamical inhomogeneities are calculated in the same way as before, then

\[
\left( \frac{\Delta \omega}{\omega} \right)_{is} = \left( \frac{n^4 \beta^2 \kappa T \beta}{V_{\text{eff}}} \right)^{1/2} = \left( \frac{3 \lambda^4 \sigma_{is}}{8 \pi n^4 V_{\text{eff}}} \right)^{1/2}. \tag{34}
\]

is in agreement with qualitative estimates in Refs. 5 and 11. Effective volume of the most interesting TE\(_{11}\) mode can be calculated according to the formula

\[
V_{\text{eff}} = 2.3n^{-7/6}a^{11/6}V_0^{7/6}. \tag{35}
\]

In this way, for TE\(_{11}\) mode in fused-silica microsphere we obtain

\[
\left( \frac{\Delta \omega}{\omega} \right)_{is} = 5 \times 10^{-7} \ \mu m^{3/2}a^{7/12}. \tag{36}
\]

If \( l \neq m \), the following asymptotic approximation is valid:

\[
V_{\text{eff,lm}} = V_{\text{eff,0}}(1 + 0.5 \sqrt{l - m} - 0.5). \tag{37}
\]

Now let us analyze the case of mode splitting that is due to the surface inhomogeneities. From Eqs. (25) and (8) after averaging,

\[
\beta_{ss}^2 = \frac{\omega_0^2}{4n^4} \frac{\pi B^2 \sigma^2 |e|^4}{V_{\text{eff}}} \int |e(r)|^4 dr, \tag{38}
\]

or for TE\(_{11}\) mode in fused silica microsphere,

\[
\left( \frac{\Delta \omega}{\omega} \right)_{ss} = \frac{1.1 \sigma B}{\lambda^{1/4} a^{7/4}}. \tag{39}
\]

It is easy to show that for the measured size of surface inhomogeneities,\(^3\) the above expressions give a substantially lower level of coupling between modes than internal inhomogeneities [formula (36)].

Explicit comparison of the theoretical prediction for mode splitting \( \Delta \omega \omega \) with experimental observations requires independent assessment of bulk scattering \( \sigma_{is} \) in the material of particular sphere, as well as knowledge of the type of excited mode for correct evaluation of \( V_{\text{eff}} \).

Previous experimental reports did not have enough data for such a comparison. However, we consider the calculation results in reasonable—order of magnitude—agreement with the values of splitting observed in experiments.\(^6,10,11\)

Indeed, for a smallest volume mode TE\(_{11}\) in 150-\(\mu\)m silica sphere at 633 nm, formula (36) predicts the splitting of 30 MHz—compared with 7 MHz observed in our experiment.\(^10\) The discrepancy should be attributed to a larger volume of actually excited whispering-gallery modes.

7. CONCLUSION

In conclusion we have analyzed the effect of the volumetric (Rayleigh) and surface scattering on the quality factor and mode coupling in high-\(Q\) optical microsphere cavities. It is shown that the Rayleigh scattering has to be largely suppressed for whispering-galley modes in microspheres compared with plane-wave propagation in continuous medium, and also surface scattering losses have to be significantly altered. This reduction of scattering losses is due to restrictions imposed on permitted scattering angles by cavity confinement (i.e., discrete spectrum and very high \( Q \) of eigenmodes). As a result, earlier estimates of the fundamental limit for the \( Q \) factor have to be revisited: \( Q = 10^{12} \) is shown to be feasible in few-millimeter-size fused-silica spheres, if surface hydration problem is overcome. Modified expressions for surface scattering losses confirm the linear reduction of loss with increasing sphere diameter\(^3,4,7\) and the \( \lambda^{3/2} \) dependence of the related limit of \( Q \) on the wavelength.

We have analyzed in detail the manifestation of scattering in the form of mode coupling, the main observable effect being the resonance intracavity backscattering—coupling within the pair of initially degenerate counter-propagating WG modes. Explicit expressions and numerical estimates are obtained for the parameters of resulting mode doublets (in frequency domain), and in addition the problem of resonance reflection is solved for a system of microsphere and a traveling-wave evanescent coupler device. It is shown, in particular, that regimes with as much as 100% resonance reflection are feasible with a mode-matched coupler and appropriately tailored coupling strength—important for laser frequency locking applications.

ACKNOWLEDGMENTS

This research was supported in part by the Russian Foundation for Basic Research (grant 96-15-96780). V. S. Ilchenko participated in this work as a faculty member of Moscow University, before accepting a National Research Council senior research associateship with the NASA Jet Propulsion Laboratory.

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