Reducing the mirrors coating noise in laser gravitational-wave antennae by means of double mirrors

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Recent researches show that the fluctuations of the dielectric mirrors coating thickness can introduce a substantial part of the future laser gravitational-wave antennae total noise budget. These fluctuations are especially large in the high-reflectivity end mirrors of the Fabry-Perot cavities which are being used in the laser gravitational-wave antennae.

We show here that the influence of these fluctuations can be substantially decreased by using additional short Fabry-Perot cavities, tuned in anti-resonance instead of the end mirrors.

I. INTRODUCTION

One of the basis components of laser gravitational-wave antennae [1–3] are high-reflectivity mirrors with multilayer dielectric coating. Recent researches [4–13] have shown that fluctuations of the coating thickness produced by, in particular, Brownian and thermoelastic noise in a coating, can introduce substantial part of the total noise budget of the future laser gravitational-wave antennae. For example, estimates, done in [9] show that the thermoelastic noise value can be close to the Standard Quantum Limit (SQL) [14] which corresponds to the sensitivity level of the Advanced LIGO project [3] or even can exceed it in some frequency range.

For this reason it was proposed in [15] to replace end mirrors by coatingless corner reflectors. It was shown in this article that by using these reflectors, it is possible, in principle, to obtain sensitivity much better than the SQL. However, the corner reflectors require substantial redesign of the gravitational-wave antennae core optics and suspension system.

At the same time, the value of the mirror surface fluctuations depends on the number of dielectric layers which form the coating. It can be explained in the following way. The most of the light is reflected from the first couple of the layers. At the same time, fluctuations of the mirror surface are created by the thickness fluctuations of all underlying layers, and the larger is the layers number, the larger is the surface noise.

Therefore, the surface fluctuations are relatively small for the input mirrors (ITM) of the Fabry-Perot cavities of the laser gravitational-wave antennae with only a few coating layers and $1 - R \sim 10^{-2}$ ($R$ is the mirror power reflectivity), and is considerably larger for the end mirrors (ETM) with coating layers number $\sim 40$ and $1 - R \lesssim 10^{-5}$.

In this paper another, less radical way of reducing the coating noise, exploiting this feature, is proposed. It is based on the use of an additional short Fabry-Perot cavity instead

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FIG. 1: Schematic layout of a Fabry-Perot cavity with double mirror system instead of the end mirror: ITM and IETM are similar moderate reflective mirrors; EETM is a high-reflective one.

FIG. 2: The double reflector based on a single mirror.

of the end mirror (see Fig. 1). It should be tuned in anti-resonance, \( i.e \) its optical length \( l \) should be close to \( l = (N + 1/4)\lambda \), where \( \lambda \) is a wavelength. The back side of the first mirror have to have a few layers of an antireflection coating.

It can be shown that in this case reflectivity of this cavity will be defined by the following equation:

\[
1 - \mathcal{R} \approx \frac{(1 - R_1)(1 - R_2)}{4},
\]

(1)

where \( R_{1,2} \) are the reflectivities of the first (EETM on Fig. 1) and the second (IETM) mirrors. Phase shift in the reflected beam produced by small variations \( y \) in position of the second mirror reflecting surface relative to the first one will be equal to

\[
\phi \approx \frac{1 - R_1}{4} \times 2ky,
\]

(2)

where \( k = 2\pi/\lambda \) is a wave number. It is supposed for simplicity that there is no absorption in the first mirror material; more general formulae are presented below.

It follows from these formulae that the first mirror can have a moderate value of reflectivity and, therefore, a small number of coating layers. In particular, it can be identical to the input mirror of the main Fabry-Perot cavity (ITM). At the same time, influence of the coating noise of the second (very-high-reflective) mirror will be suppressed by a factor of \( (1 - R_1)/4 \), which can be as small as \( \sim 10^{-2} \div 10^{-3} \).

In principle, another design of the double reflector is possible, which consists of one mirror only, see Fig. 2. Both surfaces of this mirror have to have reflective coatings: the thin one on the face side and the thick one on the back side. In this case the additional Fabry-Perot
cavity is created _inside_ this mirror. However, in this case thermoelastic fluctuations of the the back surface coating will bend the mirror and thus will create unacceptable large mechanical fluctuations of the face surface. Estimates show that using this design, it possible to reduce the face surface fluctuations by factor $\sim 3$ only [16]. So the design with two _mechanically isolated_ reflectors only will be considered here.

In the next section more detail analysis of this system is presented.

### II. ANALYSIS OF THE DOUBLE-MIRROR REFLECTOR

The rightmost part of Fig. 1 is presented in Fig. 3, where the following notation is used:
- $a, b$ are the amplitudes of the incident and reflected waves for the first mirror, respectively;
- $a_0, b_0$ are the amplitudes of the waves traveling in the left and right directions, respectively, just behind the first mirror coating;
- $a_1, b_1$ are the same for the waves just behind the first mirror itself;
- $a_2, b_2$ are the amplitudes of the incident and reflected waves for the second mirror, respectively.

These amplitudes satisfy the following equations:

\[
\begin{align*}
a_0 &= -R_1 b_0 + iT_1 a, \\
a_1 &= T_0 a_0 + A_1 n_a, \\
a_2 &= \theta a_1, \\
b &= -R_1 a + iT_1 b_0, \\
b_0 &= T_0 b_1 + A_0 n_b, \\
b_1 &= \theta b_2, \\
b_2 &= -R_2 a_2 + A_2 n_2, \\
\end{align*}
\]

where:
- $n_a, n_b, n_2$ are independent zero-point oscillations generated in the first ($n_a, n_b$) and the second ($n_2$) mirrors;
- $\theta = e^{i k l_1}$, where $l_1$ is the distance between the first mirror back surface and the second mirror;
\( R_1 \) and \( iT_1 \) are the amplitude reflectivity and transmittance of the first mirror coating, respectively, \( R_1^2 + T_1^2 = 1 \);

\( T_0 \) and \( A_0 \) are the amplitude transmittance and absorption of the first mirror bulk, respectively, \( |T_0|^2 + A_0^2 = 1 \);

\( R_2 \) and \( A_2 \) are the amplitude reflectivity and absorption of the second mirror, respectively, \( R_2^2 + A_2^2 = 1 \).

\( R_1, T_1, A_0, R_2, A_2 \) are real values; \( T_0 \) is a complex one, its argument corresponds to the phase shift in the first mirror bulk.

Here we do not consider absorption in the first mirror coating for two reasons: (i) it is relatively small and (ii) it exists both in traditional one-mirror reflectors and in the one considered here, and the main goal of this short article is to emphasize the differences between these two types of reflectors.

We also suppose that the mirrors move rather slowly:

\[
\frac{dl}{dt} \ll \frac{l}{c}.
\]  

In the case of the gravitational-wave signal characteristic frequencies \( \Omega \lesssim 10^3 \text{ s}^{-1} \) and relatively short length \( l \lesssim 1 \text{ m} \) this inequality is fulfilled pretty well.

It follows from equations (3) that the reflected beam amplitude is equal to

\[
b = \frac{(R_2T_0^2\theta^2 - R_1)a - iR_2A_0T_0T_1\theta^2n_a + iA_2T_0T_1\theta n_2 + iA_0T_1n_b}{1 - R_1R_2T_0^2\theta^2}.
\]  

(5)

This solution can be presented in the following form:

\[
b = \tilde{R}a + An,
\]  

(6)

where

\[
\tilde{R} = \frac{R_2T_0^2\theta^2 - R_1}{1 - R_1R_2T_0^2\theta^2}
\]  

is the equivalent complex reflection factor for the scheme considered,

\[
A = \frac{T_1\sqrt{1 - R_2^2|T_0|^4}}{|1 - R_1R_2T_0^2\theta^2|}
\]  

(8)

is its equivalent absorption factor, and

\[
n = \frac{1}{A} \cdot \frac{-iR_2A_0T_0T_1\theta^2n_a + iA_2T_0T_1\theta n_2 + iA_0T_1n_b}{1 - R_1R_2T_0^2\theta^2}
\]  

(9)

is the sum noise normalized as zero-point fluctuations.

As mentioned above, this system should be tuned in anti-resonance:

\[
l \equiv \frac{1}{k} \arg T_0\theta = \frac{\pi}{k} \left( N + \frac{1}{2} \right) + y,
\]  

(10)

where \( N \) is an integer and \( y \ll \lambda \). In this case

\[
T_0\theta = i(-1)^N |T_0|e^{iky} \approx i(-1)^N(1 + iky),
\]  

(11)
and

\[ \tilde{R} \approx -Re^{i\phi}, \quad (12) \]

where

\[ R = 1 - \frac{(1 - R_1)(1 - R_2|T_0|^2)}{1 + R_1 R_2|T_0|^2}, \quad (13) \]

and

\[ \phi \approx \frac{2R_2|T_0|^2 T_1^2}{(R_2|T_0|^2 + R_1)(1 + R_1 R_2|T_0|^2)} ky \quad (14) \]

is the phase shift produced by the deviation \( y \) in the distance \( l \).

Suppose that factors \( T_1, A_0, A_2 \) are small. In this case

\[ 1 - R \approx \frac{(1 - R_1)(1 - R_2 + A_0^2)}{2}, \quad (15) \]

\[ \phi \approx (1 - R_1)ky. \quad (16) \]

Using power reflection and absorption factors instead of the amplitude ones:

\[ R = R^2, \quad (17) \]
\[ R_{1,2} = R_{1,2}^2, \quad (18) \]
\[ A_0 = A_0^2, \quad (19) \]

equations (15), (16) can be rewritten as follows:

\[ 1 - R \approx \frac{(1 - R_1)(1 - R_2 + 2A_0)}{4}, \quad (20) \]

\[ \phi \approx \frac{1 - R_1}{2} ky. \quad (21) \]

III. CONCLUSION

The main goal of this short article is just to claim the idea, so the detailed design of the additional cavity is not presented here. However, the following important topics have to be discussed in brief.

The first one concerns the optimal value of the \( \text{IETM} \) mirror reflectivity. The smaller is \( 1 - R_1 \), the larger is suppression factor for the \( \text{EETM} \) mirror surface noises; at the same time, the larger is the \( \text{IETM} \) mirror coating noise. The rigorous optimization requires exact knowledge of the coating noise dependence on the coating layers number.

A crude estimate based in the exponential dependence of the \( \text{IETM} \) mirror transmittance \( T_1 \approx 1 - R_1 \) on the coating layers number gives that the optimal transmittance value is relatively large, \( T_1 \approx 10^{-1} \). On the other hand, smaller values of the \( \text{IETM} \) mirror transmittance, down to the input (ITM) mirror transmittance \( T_{\text{ITM}} \) are also acceptable. Therefore, identical ITM and IETM mirrors can be used.
In the Advanced LIGO interferometer, the input mirrors transmittance will be equal to $T_{\text{ITM}} \approx 5 \times 10^{-3}$, and its bulk absorption will be equal to $A_{\text{ITM}} \approx 10^{-5}$ [17]. Using such mirror as an IETM mirror in the scheme proposed in this article, and mirror with commercially available value of $1 - R_2 \approx 10^{-5}$ as an EETM mirror, it is possible to create a double-mirror reflector with $1 - R < 10^{-6}$ and suppression factor for the EETM surface fluctuations $\frac{1 - R_1}{4} \approx 10^{-3}$.

The second issue concerns the optical power circulating through the IETM mirror. It is easy to show using equations (3), that it is $\frac{1}{4 (1 - R_1)} \sim 10^3$ times smaller than the power circulating in the main cavities. In the Advanced LIGO topology, it will be approximately equal to the power circulating through the ITM mirrors and the beamsplitter (about 1 KW).

It is necessary to note also that $y$ in the calculations presented above includes not only coating noise of the EETM mirror but all possible kinds of its surface fluctuations, including ones caused by Brownian and thermoelastic fluctuations in this mirror bulk, Brownian fluctuation in its suspension, seismic noise as well as the mirror quantum fluctuations. This feature simplifies greatly the EETM mirror design because the requirements for all these noise sources can be reduced by a factor of $(1 - R_1)/4$.

In particular, the SQL value $\sqrt{\frac{\hbar}{m\Omega^2}}$ for this mirror ($m$ is its mass and $\Omega$ is the observation frequency) can be larger by a factor of $\left(\frac{1 - R_1}{4}\right)^{-1}$. Therefore, its mass can be, in principle, $\left(\frac{1 - R_1}{4}\right)^{-2} \sim 10^6$ times smaller than for the main (ITM and IETM) mirrors. Of course, such a small mirror hardly can be used in the real interferometer. This estimates shows only that the quantum noise does not impose any practical limitation on the EETM mirror mass.

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[16] S. P. Vyatchanin, private communication.
[17] www.ligo.caltech.edu/AdvLIGO